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Appendix A from J.-F. Le Galliard et al., "Adaptive Evolution of Social Traits: Origin, Trajectories, and Correlations of Altruism and Mobility"

(Am. Nat., vol. 165, no. 2, p. 206)

Population Dynamics

We consider a social network comprising a large number of homogeneous sites occupied by a population of mutants, called y, and residents, denoted by x. A mutant y located at a site z on the network experiences the following birth, death, and movement rates:

$$\begin{aligned} b_y(z) &= \left(b + \sum_{j=x,y} \phi u_j n_{j|y}(z) - C(m_y, u_y)\right)\phi n_{0|y}(z), \\ d_y(z) &= d, \\ m_y(z) &= m\phi n_{0|y}(z). \end{aligned} \quad \text{(A1)}$$

To derive the dynamics of the mutant's population size, we average birth and death rates described in equation (A1) over all sites of the network occupied by the mutant, which gives

$$\frac{dN_y}{dt} = [(b - C(m_y, u_y))\phi E(n_{0|y}(z)) - d]N_y + \sum_{j=x,y} \phi^2 u_j \sum_z n_{j|y}(z) n_{0|y}(z), \quad \text{(A2a)}$$

where $E(n_{0|y}(z))$ is the network average of the number of empty sites neighboring a site occupied by a mutant. The third term in equation (A2a) is a product between random variables describing alternative neighborhoods of a mutant individual. Assuming a multinomial probability distribution of sites and independence between the neighborhoods of pairs of sites (Morris 1997), we have

$$\sum_z n_{j|y}(z) n_{0|y}(z) = N_y n(n-1) q_{j|y} q_{0|y}, \quad \text{(A2b)}$$

where $q_{k|y}$ is the average local frequency of type k sites neighboring a mutant. The dynamics of the mutant's population size is then given by

$$\frac{dN_y}{dt} = \left\{\left[b + \sum_{j=x,y} (1-\phi) u_j q_{j|y} - C(m_y, u_y)\right] q_{0|y} - d\right\} N_y = \lambda_y N_y, \quad \text{(A2c)}$$

which involves the configurations of pairs of sites. A closed system describing the pair dynamics is obtained by Le Galliard et al. (2003) from the bookkeeping of all events affecting pairs of sites:

$$\frac{dN_{0y}}{dt} = (\alpha'_y q_{0|0} - \beta_y - \delta_y)N_{0y} + \delta_x N_{xy} + \delta_y N_{yy},$$

$$\frac{dN_{yy}}{dt} = 2\beta_y N_{0y} - 2\delta_y N_{yy}, \qquad \text{(A3)}$$

$$\frac{dN_{xy}}{dt} = (\alpha_x + \alpha'_y q_{x|0})N_{0y} - (\delta_x + \delta_y)N_{xy},$$

where α_i is the average per capita input rate of a type i individual into a type 0j pair with $j \neq i$ ($\alpha_i = \alpha'_i q_{i|0}$), β_i is the average per capita input rate of a type i individual into a type 0i pair, and δ_i is the average per capita output rate of a type i individual from a type ij pair (following van Baalen and Rand 1998; see also app. 2 in Le Galliard et al. 2003).

In general, a resident population converges to a unique stable equilibrium spatial structure, which is described in appendix 3 of Le Galliard et al. (2003). The nontrivial population equilibrium is characterized by $\bar{q}_{x|x}$, which satisfies the quadratic equation $[b + u_x(1 - \phi)\bar{q}_{x|x} - C(u_x, m_x)](1 - \bar{q}_{x|x}) - d = 0$, and by $\bar{q}_{0|0} = \delta_x/\alpha'_x$. If b is sufficiently larger than d, the resident population is nonviable when $\Delta < 0$, where Δ denotes the discriminant of the quadratic equation.